Analysis and Design of Slot-Coupled Directional Couplers Between Double-Sided Substrate Microstrip Lines

Man-Fai Wong, Victor Fouad Hanna, *Senior Member, IEEE*, Odile Picon, *Member, IEEE*, and Henri Baudrand, *Senior Member, IEEE*

***Abstract*— This paper proposes to study the characteristics of a slot-coupled directional coupler between two microstrip lines coupled through a rectangular slot in the common ground plane. Firstly, conformal mapping techniques are used to obtain analytic closed-form expressions for the coupler even and odd-mode impedances and propagation constants for any coupler configuration. Secondly, a full-wave analysis is performed using the spectral domain approach to determine the dispersion properties of coupler parameters. Theoretical and experimental results for a 10 dB coupler at 10 GHz are presented.**

I. Introduction

A NEW slot-coupled directional coupler between two microstrip lines coupled through a rectangular slot in the common ground plane (Fig. 1) was proposed for the first time by Tanaka *et al.* [1]. This coupler can find important applications in the design of beam forming networks and multiport amplifiers through the use of this new coupler in the construction of planar multiport directional couplers without the necessity of using microstrip cross-overs. This coupler enables both tight and loose coupling values to be achieved. Here, firstly we propose to use conformal mapping techniques to obtain analytic closed-form expressions for its quasi-static even and odd-mode parameters. Results using these expressions are in a very good agreement with those obtained by Tanaka *et al.* using a heavy numerical method (finite element method) always for the quasi-static case. Secondly, a full wave analysis using spectral domain approach [2], [3] is performed to obtain the dispersion characteristics of the coupler's even and odd-mode parameters. Finally, a 10 dB coupler is realized and a comparison of theoretical and experimental results for this coupler parameter is presented.

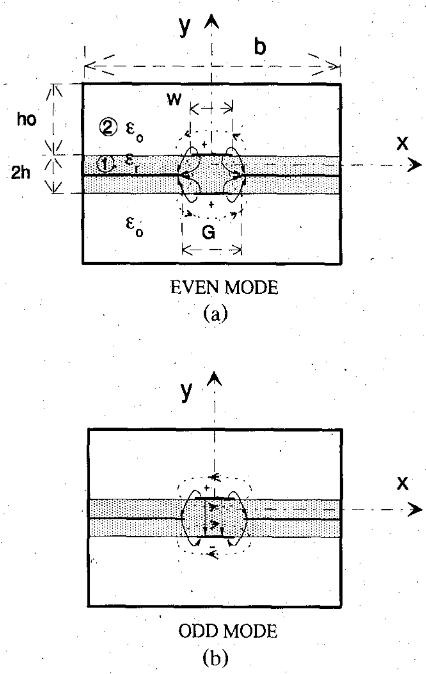

Fig. 1. Coupler's electric (full line) and magnetic (dashed line) field distribution for (a) even-mode and (b) odd-mode.

II. Quasi-Static Analysis

In this quasi-static analysis, we assume the ground planes to be infinitely wide and the strips to have negligible thickness (Fig. 2). Also, we assume that the air-dielectric interfaces like AD and A′D′ can be dealt with as though perfect magnetic walls were present in them (i.e., enforcing Neuman boundary conditions). This assumption can be easily verified for small gaps, however it is noticed

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M. F. Wong, V. Fouad Hanna, and O. Picon are with CNET-PAB-STS, 38-40 rue du General Leclerc, 92131 Issy-les-Moulineaux, France.
H. Baudrand is with Lab. de Microondes, ENSEEIHT, 2 rue Charles Camichel, 31071 Toulouse, France.
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that it leads to adequately accurate results for fairly large ones.

It can be easily seen that the existence of the slot in the common ground plane does not affect the coupler odd-mode characteristics which are similar to those for a shielded microstrip line.

The even and odd-mode coupler impedances, Z_{oe} and Z_{oo}, respectively, are calculated using conformal mapping techniques to determine the coupler capacitance per unit length for even and odd-modes. For each mode, the overall capacitance per unit length, C_T, can be considered as the sum of the coupler capacitance per unit length, C_1, for the space bounded by the upper shielding and the upper microstrip half plane which is filled by air and the capacitance per unit length, C_2, for the space bounded by the upper microstrip and the common ground plane which is filled by the dielectric. To obtain these capacitances for the even mode, C_{1e} and C_{2e}, the sequence of conformal transformations shown in Fig. 3 is utilized noticing that the line BB′ is a magnetic wall. The goal in the two cases is to map the original boundary value problem in the z plane into rectangle in a final t plane. Hence the total even-mode capacitance per unit length can be put in the form

$$C_{Te} = C_{1e} + C_{2e} \tag{1}$$

$$C_{1e} = 2\epsilon_o K(k_2)/K'(k_2) \tag{2}$$

$$C_{2e} = 2\epsilon_o \epsilon_r K'(k_1)/K(k_1) \tag{3}$$

where

$$k_2 = \tanh(\pi W/4h_o) \tag{4}$$

$$k_1 = \sqrt{\frac{\sinh^2(\pi G/4h)}{\sinh^2(\pi G/4h) + \cosh^2(\pi W/4h)}} \tag{5}$$

and where $K(k)$ is the complete elliptic integral of the first kind and $K'(k) = K(k')$, $k' = \sqrt{1-k^2}$. Thus, the relative effective dielectric constant and the characteristic impedance for the coupler even mode can be given by

$$\epsilon_{\text{eff}_e} = \frac{C_{Te}}{C_{Te}(\epsilon_r \to 1)} = \left(\epsilon_r \frac{K'(k_1)}{K(k_1)} + \frac{K(k_2)}{K'(k_2)}\right) \Bigg/ \left(\frac{K'(k_1)}{K(k_1)} + \frac{K(k_2)}{K'(k_2)}\right) \tag{6}$$

$$Z_{oe} = 60\pi(\epsilon_{\text{eff}_e})^{-1/2}\left[\frac{K'(k_1)}{K(k_1)} + \frac{K(k_2)}{K'(k_2)}\right]^{-1}. \tag{7}$$

It can be seen that the expressions given in (6) and (7) are simple analytic closed form ones that can be calculated using the simple formulas of Hilberg [4] for the ratio $K(k)/K'(k)$.

In the same manner we can examine the coupler odd-mode characteristics noticing the line BB′ is an electric wall in this case. So the capacitance C_{1o} and C_{2o} are obtained in a similar way to that utilized for obtaining C_{1e}. So we can write their values as follows:

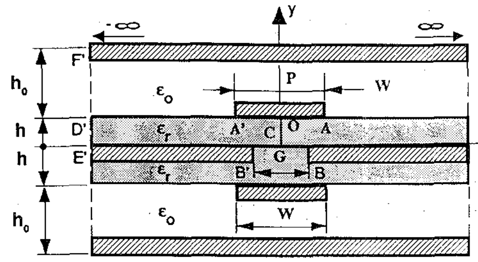

Fig. 2. Coupler model for quasi-static analysis.

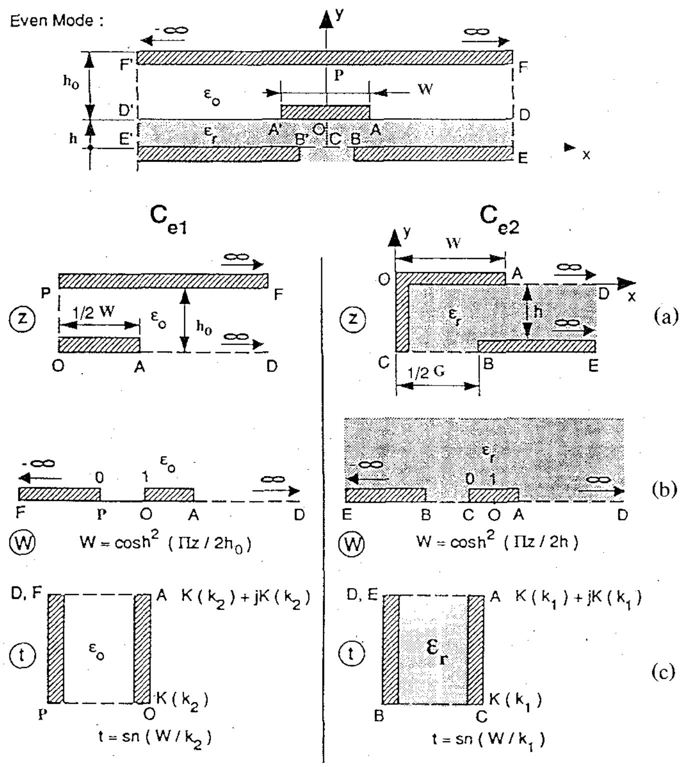

Fig. 3. Conformal transformations for obtaining the total coupler even-mode capacitance per unit length.

$$C_T = C_{1o} + C_{2o} \tag{8}$$

$$C_{1o} = 2\epsilon_o K(k_4)/K'(k_4) \tag{9}$$

$$C_{2o} = 2\epsilon_o \epsilon_r K(k_3)/K'(k_3) \tag{10}$$

where

$$k_3 = \tanh(\pi W/4h) \tag{11}$$

$$k_4 = \tanh(\pi W/4h_o). \tag{12}$$

Hence, the coupler odd-mode relative effective dielectric constant and the odd-mode characteristic impedance can be given by the following simple analytic closed-form

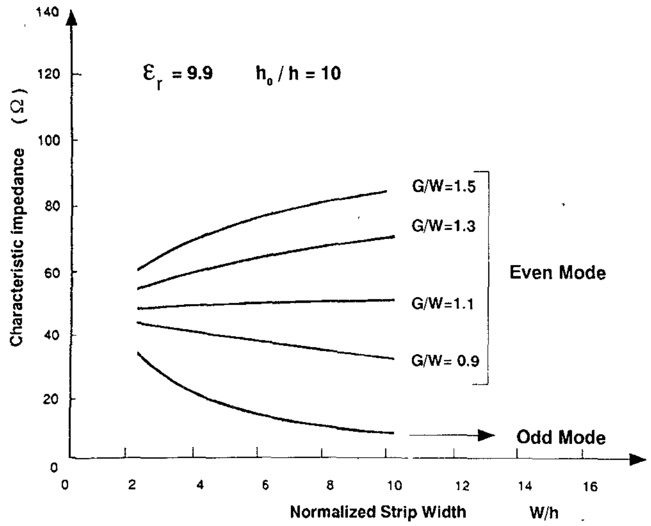

Fig. 4. Variation of the coupler even and odd-mode characteristic impedances as a function of the normalized strip width for the indicated coupler parameters.

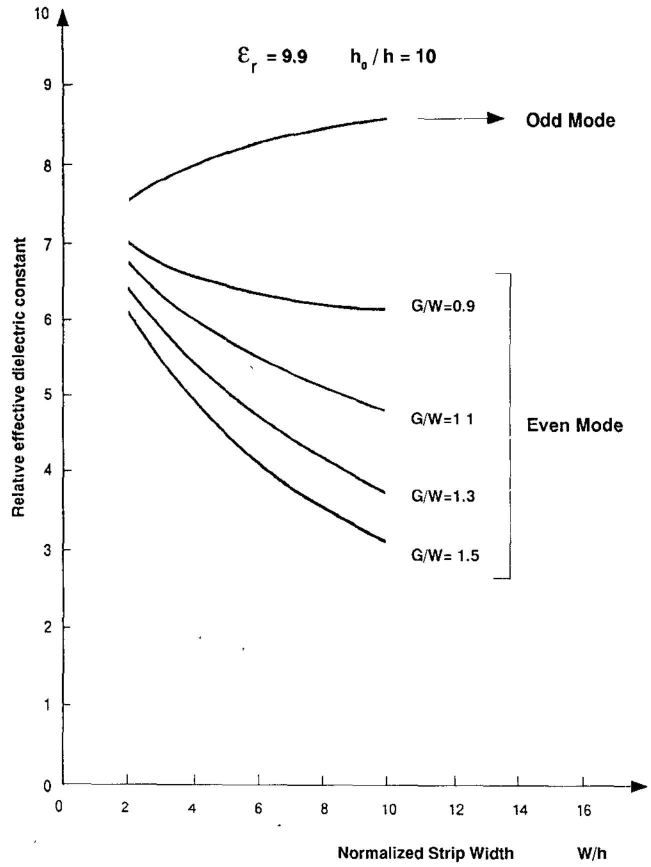

Fig. 5. Variation of the coupler even and odd-mode relative effective dielectric constants as a function of the normalized strip width for the indicated coupler parameters.

expressions:

$$\epsilon_{\mathrm{eff}_o}=\left(\epsilon_r\frac{K(k_3)}{K'(k_3)}+\frac{K(k_4)}{K'(k_4)}\right)\Big/\left(\frac{K(k_3)}{K'(k_3)}+\frac{K(k_4)}{K'(k_4)}\right) \tag{13}$$

$$Z_{oo}=60\pi(\epsilon_{\mathrm{eff}_o})^{-1/2}\left[\frac{K(k_3)}{K'(k_3)}+\frac{K(k_4)}{K'(k_4)}\right]^{-1}. \tag{14}$$

Figs. 4 and 5 give examples of design curves for the shielded slot-coupled directional coupler that were evaluated using the above obtained analytic expressions. Fig. 6 gives the variation of the coupling coefficient K as a function of the coupler geometry.

Fig. 7 gives a comparison between our results using conformal mapping techniques and those of [1] using the finite element method for a quasi-static solution. A good agreement is noticed with a deviation which does not exceed 5 percent for the even-mode characteristics and 1 percent for the odd-mode ones.

III. Full Wave Analysis

In order to simplify the analysis without losing generality, we assume that the coupler is inserted in a closed box of width b as shown in Fig. 1 where the coordinate axes x and y are also defined. Also in this case, the existence of the slot in the common ground plane does not affect the coupler odd-mode characteristics which are similar to those for a closed microstrip line which has already been characterized by the spectral domain approach [2]. Hence, only the even mode analysis is given here in detail. Again the plane containing the slot is a symmetry plane (magnetic wall for even mode), we consider only half of the structure.

The axial field components $E_{z,i}(x,y)$ and $H_{z,i}(x,y)$ in the two regions 1 and 2 are expanded in Fourier series (Fig. 1) within their domain:

$$-b/2\leqslant x\leqslant b/2.$$

The following expansions of $E_{z,i}(x,y)$ and $H_{z,i}(x,y)$ are valid:

$$\boldsymbol{E}_{z,i}(x,y)=\sum_{n=-\infty}^{\infty}\tilde{\boldsymbol{E}}_{z,i}(\alpha_n,y)e^{j\alpha_n x}$$

$$\boldsymbol{H}_{z,i}(x,y)=\sum_{n=-\infty}^{\infty}\tilde{\boldsymbol{H}}_{z,i}(\alpha_n,y)e^{j\alpha_n x} \tag{15}$$

where $\alpha_n=(2n-1)\pi/b$ and quantities with the sign ~ designate the line amplitude (i.e., the nth term of the Fourier series) associated with the space harmonic α_n.

The partial differential equations for the axial field components $E_{z,i}(x,y)$ and $H_{z,i}(x,y)$ are also Fourier expanded with respect to x; ordinary differential equations are derived for the n th line amplitudes $\tilde{E}_{z,i}(x,y)$ and $\tilde{H}_{z,i}(x,y)$, respectively.

Extensive algebraic manipulations of problem boundary conditions yield functional equations relating the nth line amplitude of the tangential electric field on the slot plane and the current on the strip plane to the current on the slot plane and the tangential electric field on the strip plane through the Fourier transformed Green's function

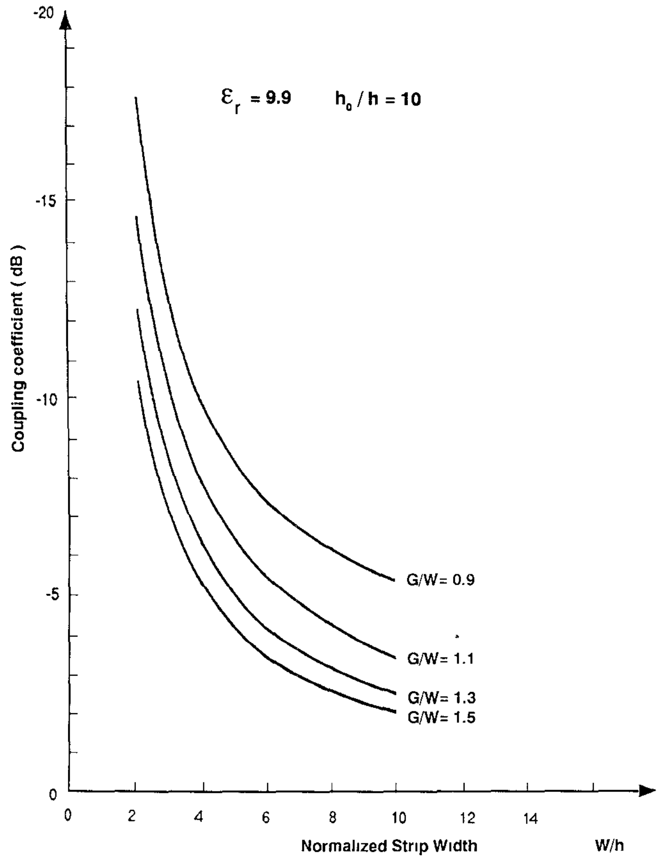

Fig. 6. Variation of the coupler coupling coefficient as a function of the normalized strip width for the indicated coupler parameters.

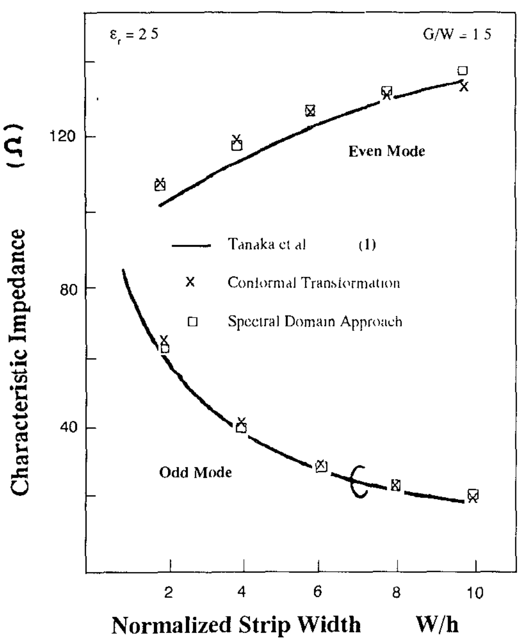

Fig. 7. Variation of the coupler even and odd-mode characteristic impedances as a function of the normalized strip width for the indicated coupler parameters: [1] (full line), conformal transformation analysis (cross marker) and spectral domain analysis (square marker).

matrix which takes the form:

$$\begin{pmatrix} \tilde{J}_x(\alpha_n, -h/2) \\ \tilde{J}_z(\alpha_n, -h/2) \\ \tilde{E}_x(\alpha_n, h/2) \\ \tilde{E}_z(\alpha_n, h/2) \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{pmatrix} \cdot \begin{pmatrix} \tilde{E}_x(\alpha_n, -h/2) \\ \tilde{E}_z(\alpha_n, -h/2) \\ \tilde{J}_x(\alpha_n, h/2) \\ \tilde{J}_z(\alpha_n, h/2) \end{pmatrix}. \quad (16)$$

The numerical part is started by applying Galerkin procedure to (16). The tangential electric field and the current in the right column are expanded in terms of sets of the following basis functions:

$$E_x(x, -h/2) = \sum_{p=1}^{P} a_p e_{xp}(x, -h/2)$$

$$E_z(x, -h/2) = \sum_{q=1}^{Q} b_q e_{zq}(x, -h/2)$$

$$J_x(x, -h/2) = \sum_{r=1}^{R} c_r j_{xr}(x, h/2)$$

$$J_z(x, -h/2) = \sum_{s=1}^{S} d_s j_{zs}(x, h/2). \quad (17)$$

By using an inner product consistent with Parseval's theorem, Galerkin's procedure is directly applied to the matrix form (16) in the Fourier domain. A set of $P+Q+R+S$ homogeneous and linear equations, for which the $P+Q+R+S$ unknowns are precisely the constants a_p, b_q, c_r and d_s, is obtained.

Nontrivial solutions of this set of equations occur for zero values of its matrix determinant. The real roots (i.e., $\beta^2 > 0$) determine the propagating eigenmodes, whereas the imaginary roots (i.e., $\beta^2 < 0$) determine evanescent ones.

The electric and magnetic fields can then be calculated and the characteristic impedance is evaluated according to the following power-current definition

$$Z_c = 2P/I^2 \quad (18)$$

with

$$P = \frac{1}{2}\int_S (\vec{E} \wedge \vec{H}^*)\, d\vec{S} = \frac{1}{2}\int_S (E_x H_y^* - E_y H_x^*)\, dS \quad (19)$$

$$I = \int_{-\frac{w_t}{2}}^{\frac{w_t}{2}} J_{z,t}(x)\, dx. \quad (20)$$

We have calculated the even and odd-mode characteristic impedances for the coupler given in Fig. 7 using our dynamic spectral domain analysis at a frequency of 1 GHz. The results are plotted in Fig. 7. These results

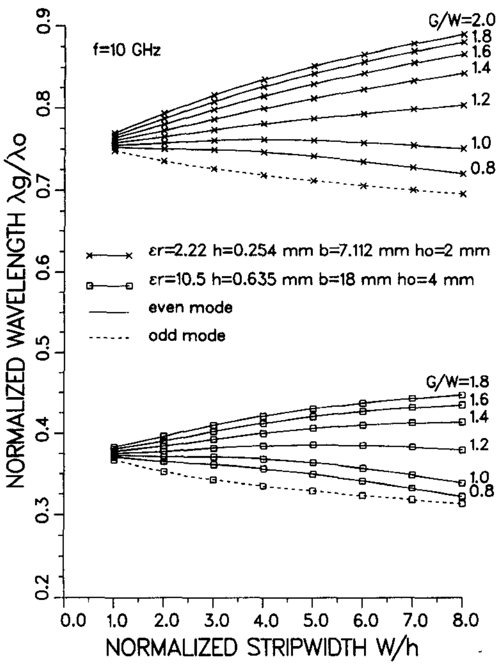

Fig. 8. Variation of the coupler even and odd-mode normalized wavelength as a function of the normalized strip width for the indicated coupler parameters given for $\epsilon_r = 2.22$ and $\epsilon_r = 10.5$.

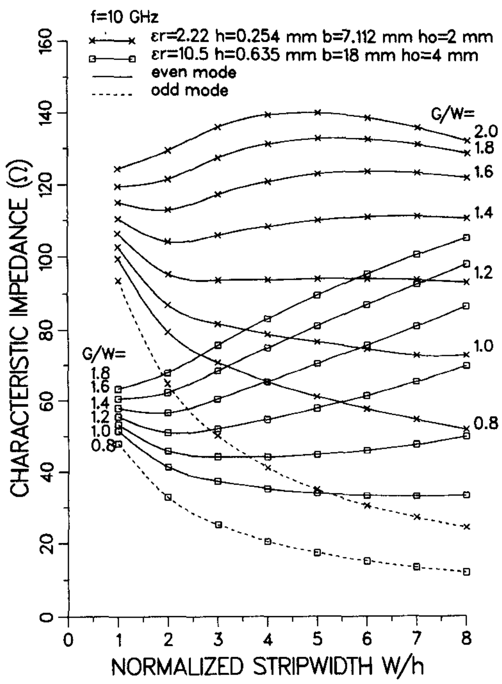

Fig. 9. Variation of the coupler even and odd-mode characteristic impedances as a function of the normalized strip width for the indicated coupler parameters given for $\epsilon_r = 2.22$ and $\epsilon_r = 10.5$.

nearly coincide with those obtained by our quasi-static analysis and have a deviation of less than 5% from those given in [1].

IV. Coupler Design

The common formulas for designing quarter wavelength coupler using symmetric lines are used:

$$K_{db} = -20\log\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right) \qquad Z_o = \sqrt{Z_{ce}Z_{co}}$$

$$l = \pi/(\beta_e + \beta_o). \tag{21}$$

The modal impedances Z_{ce}, Z_{co} and modal phase constants β_e, β_o of the fundamental mode are determined by the spectral domain analysis. The basis functions corresponding to the even fundamental mode are

$$e_x(x, -h/2) = 2x/G\left[1-(2x/G)^2\right]^{-1/2} \quad, |x| \leqslant G/2, \quad 0 \text{ elsewhere}$$

$$e_z(x, -h/2) = \sqrt{1-(2x/G)^2} \quad, |x| \leqslant G/2, \quad 0 \text{ elsewhere}$$

$$j_x(x, h/2) = 2x/W\sqrt{1-(2x/W)^2} \quad, |x| \leqslant W/2, \quad 0 \text{ elsewhere}$$

$$j_z(x, h/2) = 1/\sqrt{1-(2x/W)^2} \quad, |x| \leqslant W/2, \quad 0 \text{ elsewhere}. \tag{22}$$

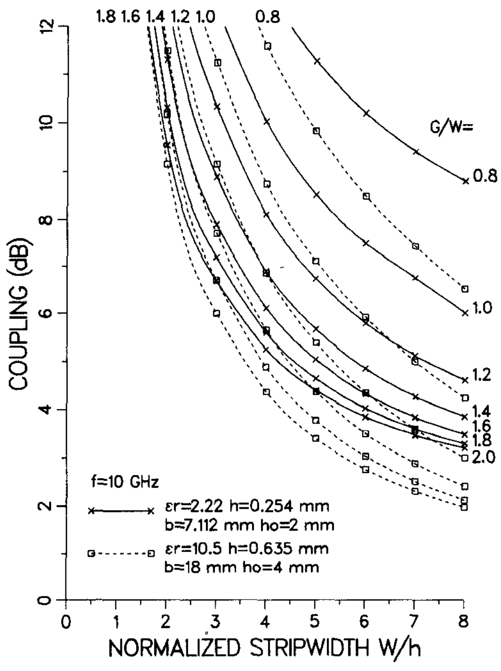

Fig. 10. Variation of the coupler coupling coefficient as a function of the normalized strip width for the indicated coupler parameters given for $\epsilon_r = 2.22$ and $\epsilon_r = 10.5$.

Figs. 8–10 show examples of design curves that give respectively the normalized wavelength, the characteristic impedance and the coupling coefficient versus the normalized strip width W/h for different G/W ratios at a frequency of 10 GHz for two types of substrate whose thickness and relative dielectric constant are either $h = 0.635$ mm and $\epsilon_r = 10.5$ or $h = 0.254$ mm and $\epsilon_r = 2.22$.

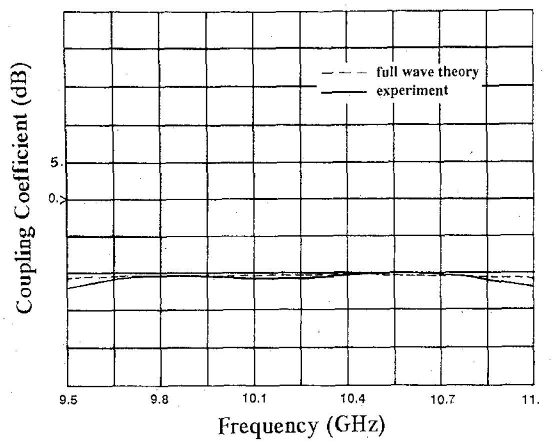

Fig. 11. Variation of the coupler coupling coefficient as a function of frequency: experiment (full line) and SDA full-wave analysis (dashed line).

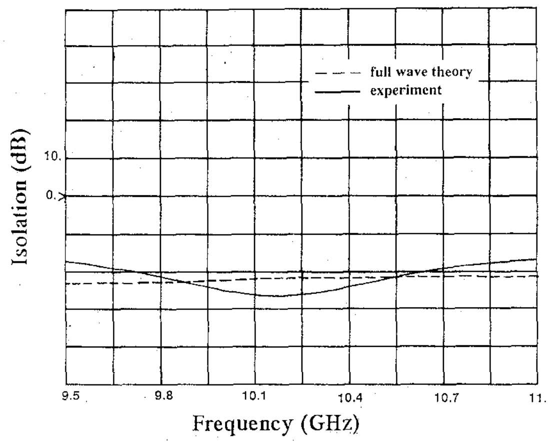

Fig. 12. Variation of the coupler isolation as a function of frequency: experiment (full line) and SDA full-wave analysis (dashed line).

V. Experimental Results

Using full-wave analysis, a 10 dB coupler is realized having the following parameters at 10 GHz:

$$Z_{ce} = 69.4\ \Omega,\ Z_{co} = 36\ \Omega,\ Z_o = 50\ \Omega,$$

$$\epsilon_r = 10.5,\ W = 1.087\ \text{mm},\ G = 2.120\ \text{mm},\ L = 2.818\ \text{mm}.$$

The measurements are performed using a conventional automatic network analyzer. The coupling and the isolation coefficients are presented in Figs. 11 and 12. Good agreement is achieved between theoretical and experimental results. The coupler bandwidth is about 10% around 10.25 GHz with a coupling coefficient of 10.2 ± 0.2 dB.

VI. Conclusion

Simple analytic closed-form expressions for shielded slot-coupled directional coupler design parameters are obtained using conformal mapping techniques. Also, for a full-wave analysis of the problem, the spectral-domain technique was shown to be very efficient to determine the dispersion properties of coupler parameters. The accuracy of this technique has been verified experimentally, and the precision of the closed-form expressions has been confirmed by comparison with results obtained by the spectral domain technique.

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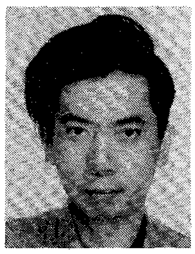


Man-Fai Wong was born in Hong-Kong on December 17, 1965. He received the Engineering diplome ("Diplome d'Ingénieur") degree in electronics from "Ecole Nationale d'Electronique, d'Electrotechnique, d'Informatique et d'Hydraulique de Toulouse," Toulouse, France in 1990. Since then he has been with the Centre National d'Etudes des Télécommunications in the application of Electromagnetics in Microwaves Group where he is presently studying toward the Ph.D. degree from Paris 7 University. His research deals with electromagnetic theory and numerical methods for solving field problems especially finite element method.

Victor Fouad Hanna (SM'87) was born in Cairo, Egypt, on November 1, 1944. He received the B.Sc. degree (honors) in electronic engineering from Cairo University, Cairo, Egypt in 1965, and the M.Sc. degree in microwave engineering from Alexandria University, Alexandria, Egypt in 1969. He received the D.Sc. degree [Doctorat ès-Sciences Physiques (doctorat d'Etat)] from "l'Institut National Polytechnique (I.N.P.)," Toulouse, France in 1975.

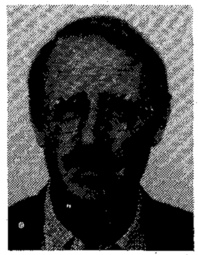
He was a Research Assistant in the National Research center, Cairo, and the Microwave Laboratory of the "I.N.P.," Toulouse, from 1965–1970 and 1970–1975, respectively. From 1975–1979 he was a Researcher in the Electrical and Electronics Engineering Laboratory, National Research Center, Cairo, engaged in research in the field of microwave theory and techniques and microwave solid state devices.

Since 1979, he has been with the Centre National d'Etudes des Télécommunications, France where he is now responsible for the group "Application of Electromagnetic Theory in Microwave and Millimeter Circuits" in its Satellite Telecommunications Systems Division. His current research interests deal with electromagnetic theory, numerical methods for solving field problems, characterization of microstrip-like transmission lines and millimeter-wave transmission lines.

Dr. Fouad Hanna has authored 90 technical papers. He has been the Chairman of the MTT-S France chapter since its creation in 1988.

Odile Picon was born in Paris, France, in 1951. She received the "Agregation of Physic" in 1976 and the D.Sc. degree (Doctorat ès-Sciences Physiques) from University of Rennes in 1988.

Since 1982, she has been a Research Engineer in the "Centre National d'Etudes des Télécommunications." Her research deals with electromagnetic theory, particularly with discontinuities on planar structures.



Henri Baudrand (M'86–SM'90) was born in France in 1939. He received the Diplome d'Ingénieur degree in electronics and the Docteur-ès-Science degree in microwaves, both from the Institut National Polytechnique of Toulouse, France, in 1962 and 1966, respectively.

Since then he has been working on the modeling of active and passive microwave circuit devices in the Electronics Laboratory of ENSEEIHT in Toulouse. Currently he is a Professor of Microwaves and is in charge of the Microwaves Research Group.